University of California, Riverside
Department of Mathematics

Midterm
Mathematics 9B - First Year of Calculus
Sample 5

Instructions: This exam has a total of 100 points. You have 50 minutes. You must show all your work to receive full credit You may use any result done in class. The points attached to each problem are indicated beside the problem. You are not allowed books, notes, or calculators. Answers should be written as $\sqrt{2}$ as opposed to 1.4142135....

1. (20 points) State the fundamental theorem of calculus, and use this theorem to find the derivative of

$$F\left(x\right) = \int_{\cos x}^{5} \frac{1}{1+u^{10}} du$$

2. Consider the two curves given by $y = x^2 - 1$ and $y = -x^2 + 2x + 3$.

   (a) (5 points) Find the points of intersection of the two curves.

   (b) (15 points) Find the area of the region bounded by both curves.

3. Let S be the region bounded by $y = 2x - x^2$ and the x-axis.

   (a) (5 points) Sketch the region S.

   (b) (15 points) Find the volume of the solid obtained by rotating the region S about the line $x = -1$.

4. (20 points) Use integrals to prove that the volume of a sphere of radius R is equal to $\frac{4}{3}\pi R^3$.

5. Compute the following integrals:

   (a) (10 points) $\int x^2 \sin\left(x^3\right) dx$

   (b) (10 points) $\int_{-\pi/4}^{\pi/4} \cos^2\left(x\right) \sin x \, dx$